

Physics Department

Advanced Higher Physics

Unit 3

Topic 2 – Circuits

Useful Websites

www.scholar.hw.ac.uk

<http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html>

2 Circuits	
Content	Notes
a) Capacitors	<p>Capacitors in dc circuits. Current and potential difference in CR circuits during charging and discharging.</p> <p>Determining the time constant for a CR circuit.</p> <p>Capacitors in ac circuits. Capacitive reactance.</p>
b) Inductors	<p>Electromagnetic induction. Investigating the factors affecting the size of the induced e.m.f. in a coil. Investigating the growth and decay of current in a dc circuit containing an inductor.</p> <p>Self inductance.</p> <p>Lenz's Law. The magnitude of the induced e.m.f.</p> <p>Determining the self inductance of a coil.</p> <p>Energy stored by an inductor.</p> <p>Inductors in ac circuits. Inductive reactance.</p> <p>Investigating the relationship between current, frequency and inductive reactance.</p>

Capacitors

In Higher Physics – Unit 3 the following topics were covered:

1. Capacitors in dc circuits.
2. Current and potential difference in CR circuits during charging and discharging.

In the space below write a summary with graphs showing how current through and potential difference across a resistor vary, when

- the size of the resistance or
- the size of the capacitance is changed.

Time Constant for Resistor / Capacitor (RC) Circuits

Show That RC Has The Units of Time

$$R = \frac{V}{I}$$

$$\Rightarrow RC = \frac{QV}{VI} = \frac{Q}{I} = \frac{It}{I} = t$$

$$C = \frac{Q}{V}$$

Definition of Decay Time (τ) for a RC Circuit

τ (*tau*) is the symbol used for time in this case.

The **time (τ) in a charging RC circuit for the voltage** across the resistor to **decay by 0.6 (60%)** of its original value is equal to **RC seconds**.

where

R = size of the resistor (ohms)

C = size of the capacitor (farads)

After **5RC seconds the voltage** across the resistor in the charging RC circuit will have **decayed to 0.01 (1%)** of its maximum value.

Example

If the maximum voltage across a $100\ \Omega$ resistor in an RC circuit where the capacitance is $500\ \text{nF}$ is given as 10V then,

Time (s) (=RC)	Total Time (s)	Initial Voltage Across R (V)	Voltage Lost (V)	New Voltage Across R (V)
$100 \times 500 \times 10^{-9} = 5 \times 10^{-5}$	5×10^{-5} (=RC)	10	$0.6 \times 10 = 6$	$10 - 6 = 4$
$100 \times 500 \times 10^{-9} = 5 \times 10^{-5}$	10×10^{-5} (=2RC)	4	$0.6 \times 4 = 2.4$	$4 - 2.4 = 1.6$
$100 \times 500 \times 10^{-9} = 5 \times 10^{-5}$	15×10^{-5} (=3RC)	1.6	$0.6 \times 1.6 = 0.96$	$1.6 - 0.96 = 0.64$
$100 \times 500 \times 10^{-9} = 5 \times 10^{-5}$	20×10^{-5} (=4RC)	0.64	$0.6 \times 0.64 = 0.384$	$0.64 - 0.384 = 0.256$
$100 \times 500 \times 10^{-9} = 5 \times 10^{-5}$	25×10^{-5} (=5RC)	0.256	$0.6 \times 0.256 = 0.1536$	$0.256 - 0.1536 = 0.1024$

As the example shows the voltage across the resistor reaches 1% of its maximum value after $5RC$ seconds.

Remember

- The voltage across the resistor decreases as the voltage across the capacitor increases
- It can also be stated that that the time (τ) for the voltage across the capacitor in a charging RC circuit to reach 0.6 of its maximum value is RC seconds and after $5RC$ seconds it will be within 0.1 of its maximum value.
- When the voltage across the capacitor is within 0.1 of its maximum value it is said to be fully charged.
- The same pattern can be applied to a discharging capacitor.

Investigating Current and Frequency in A RC Circuit.

Place a capacitor in series with a resistor and a signal generator.
Measure the current through the resistor as the frequency is changed.
Plot a graph of the results.

Graph shows that current is directly proportional to frequency

Repeat experiment, keep the frequency constant but change the size of the capacitance. Again measure the current for each value of capacitance.
Plot a graph of results.

Graph shows that current is directly proportional to capacitance

Conclusion

$$I \propto V$$

$$I \propto f$$

$$I \propto C$$

Together this gives:

$$I \propto fVC \quad \text{and the constant in the proportionality is } 2\pi$$

$$I = 2\pi fVC = V / X_C \quad \text{by comparison with } V = IR$$

where

$$\text{Capacitive Reactance } (X_C) = 1 / 2\pi fC$$

The units are ohms.

Examples

Calculate the capacitive reactance of a 100 μ F capacitor in a circuit with a 12 V a.c. supply at a frequency of 50 Hz.

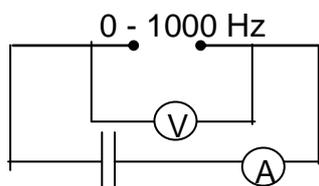
Calculate the current in a circuit consisting of a 2000 μ F capacitor connected to a 20 V supply at a frequency of 25 Hz.

Capacitors In a.c. Circuits

Aim

To show that the current in a circuit containing a capacitor is directly proportional to the frequency of the a.c. supply.

Apparatus



Method

1. Set up the circuit above.
2. Use the voltmeter to ensure that the a.c. voltage remains constant
The amplitude dial on the signal generator should be used to keep the a.c. voltage constant.
3. Vary the frequency of the supply.
4. Record the current reading associated with each frequency.
5. Plot the results of I against f .

Results

Frequency (Hz)	Current (mA)
0	
200	
400	
600	
800	
1000	

Conclusion

The best fit straight line through the origin, shows that the current I_C in a capacitive circuit is directly proportional to the frequency.

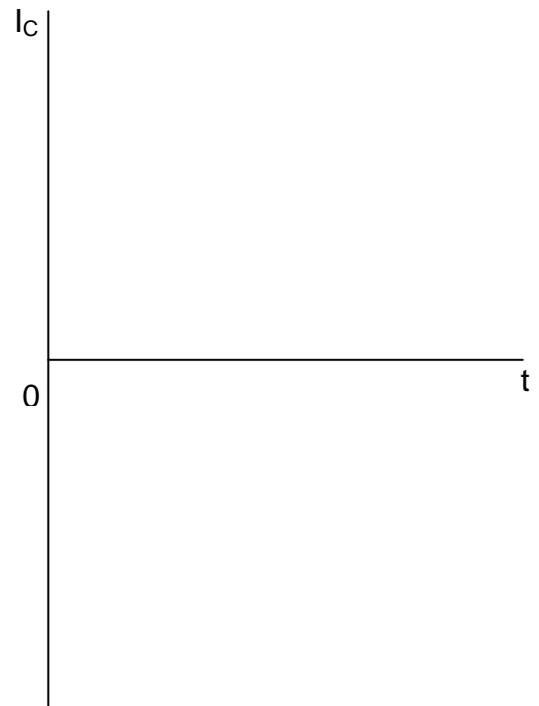
$I_C \propto f$
Graphs

Low Frequency a.c. Supply



Average I_C is smaller because the current has time to fall to a low value

High Frequency a.c. Supply



Average I_C is bigger because the current never has time to fall to a low value

As a consequence of this, if there is a series circuit containing:

- a bulb,
- an a.c. supply,
- a resistor and
- a capacitor in series then:
 - ❖ as the **frequency** is **increased** the **brightness** of the bulb will **increase**.

Uses of Capacitors

❖ Storing Energy

Camera flashgun

❖ Time Delays

Capacitors can introduce a time delay into electronic circuits due to the time they take to charge.

❖ Tuning

Capacitors are used when tuning into a radio station as part of the resonance circuit in a radio.

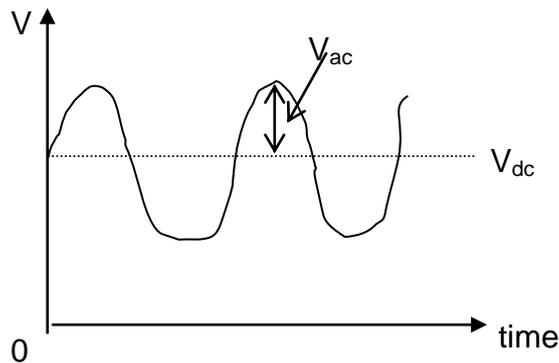
❖ Blocking d.c. but allowing a.c. to pass

A capacitor has infinite resistance to direct current (as this has zero frequency).

Once charged a capacitor will stop d.c. but a.c. will flow as it is always changing direction.

As a result a capacitor will eliminate any d.c parts of a signal.

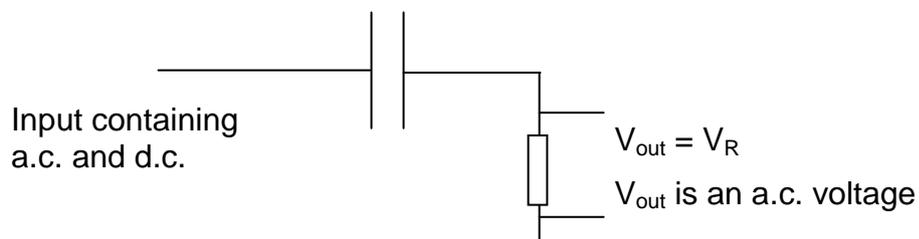
The result is that the output voltage measured across the resistor contains V_{ac} only.



This is an example of a signal which contains:

- a.c. component
 V_{ac} = the amplitude of the wave
- d.c. component
 V_{dc} = dotted line on the graph

If this input signal above is “put through” the following circuit then the output is a.c.

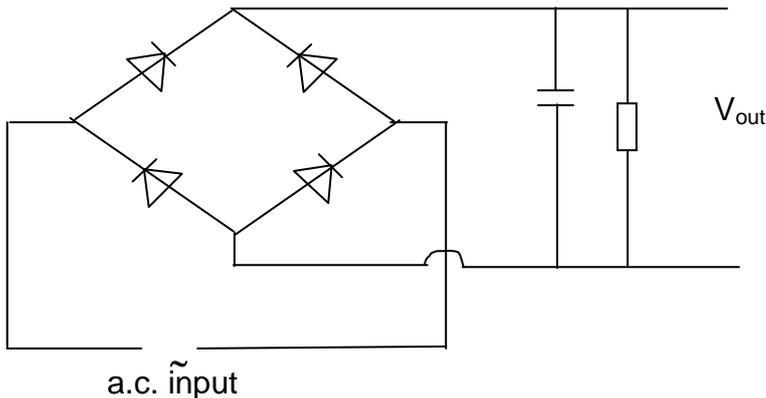


❖ **Full Wave Rectification – For Information**

This is not as bad as the title suggests!!!

Full Wave Rectification is when a number of capacitors are used to smooth a rectified d.c. signal.

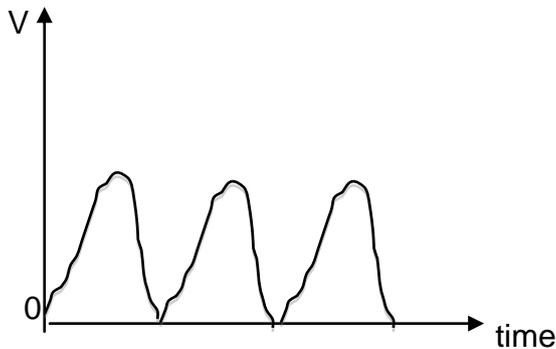
A **rectified d.c.** signal is produced when four diodes connected as shown below to produce a d.c. output from an a.c. input.



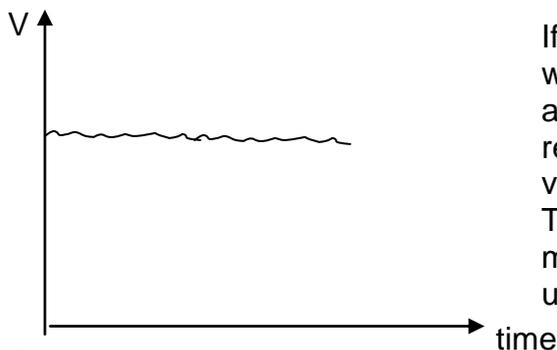
The output voltage is d.c. but it is not smooth.

If a capacitor is connected in parallel with the supply, the constant charging and discharging will reduce the rectified voltage to an almost smooth voltage with a small “ripple”.

This is illustrated below.



The diode combination produces this output. Such a circuit is known as a **Rectifier Bridge**.

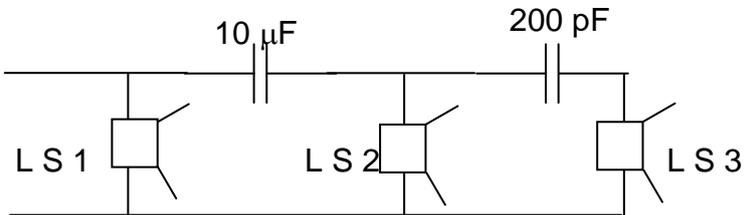


If a capacitor is connected in parallel with the supply, the constant charging and discharging will reduce the rectified voltage to an almost smooth voltage with a small “ripple”.

This is how d.c is produced from the mains a.c voltage in the power packs used in the lab.

❖ Frequency Banding For Hi-Fis

A typical loud speaker system has three separate loud speakers to transmit different noise frequency bands separately. As a result the overall sound quality is greatly improved.



Loud Speaker 1 (LS 1)

This is a large loud speaker which deals well with loud low frequency sounds.

LS 1 known as the **Woofers**.

Loud Speaker 2 (LS 2)

The $10\ \mu\text{F}$ capacitor is quite small so it has a high resistance at low frequencies. Only middle to high frequencies are passed on to LS 2.

LS 2 is known as the **Warbler**.

Loud Speaker 3 (LS 3)

The very small capacitor blocks all but the high frequency sounds from reaching LS3.

LS3 is known as the **Tweeter**

Inductors

Our present day large scale production and distribution of electricity would not be economically reasonable if the only source of electricity came from chemical sources like dry cells.

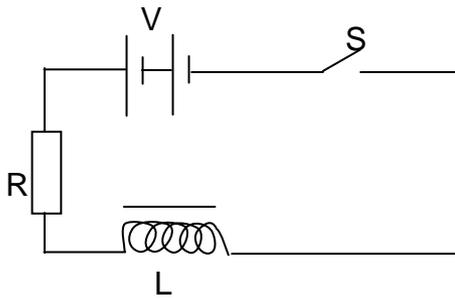
The development of electrical engineering began with the work of **Faraday** and **Henry**.

Electromagnetic induction involves the transformation of mechanical energy into electrical energy.

Points to remember from previous courses

- ☆ When a magnet is moved into or out of a coil of wire a current is induced. A current is only induced when movement is taking place.
- ☆ Whether it is the magnet or the coil that moves makes no difference to the size of the induced current...it is relative motion that is important.
- ☆ The current flows in one direction when the magnetic moves into the coil and in the other direction when the magnet moves out of the coil.
- ☆ The induced currents are produced by an induced e.m.f.
- ☆ The initial energy has come from the person pushing the magnet into the coil.
- ☆ Size of the induced e.m.f. depends on:
 - > the relative speed of the movement of the magnet and coil (faster means bigger induced e.m.f.).
 - > the strength of the magnet (the stronger the magnet the greater the induced e.m.f.)
 - > the number of turns on the coil (the greater the number of turns the greater the induced e.m.f.)
- ☆ Magnetic flux lines are used to indicate the strength and direction of the magnetic field.
- ☆ A current carrying coil is known as a solenoid
- ☆ Any metallic conductor contains “free” electrons which are not bonded to any particular atom. Therefore when an e.m.f. is applied, these electrons drift along the conductor i.e. electric current flows.
- ☆ Two famous scientists Michael Faraday and Heinrich Lentz investigated many of the concepts that are looked at in this section.

Self Inductance.

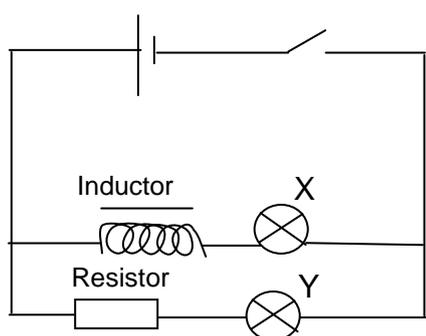


When a steady current flows, the magnetic field in and around the coil is stable.

When the current changes (by opening or closing the switch) the magnetic field changes and an e.m.f. is induced in the coil.

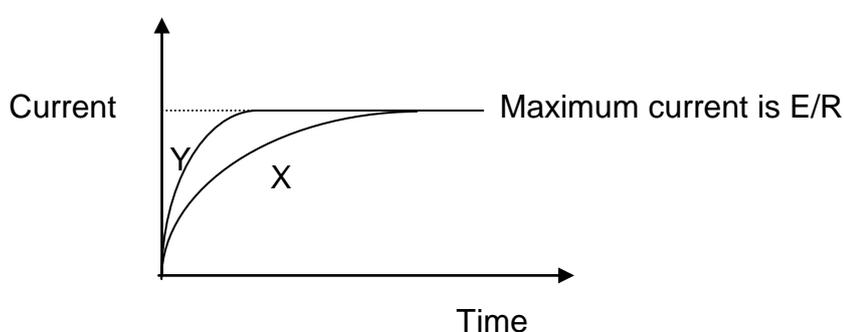
This known as the **self induced e.m.f.**

Experiment



In the circuit above, bulb Y will light before bulb X. This happens because:

- ❖ An inductor is a coil that delays the growth of the current through both bulb X and itself.



When the switch is first closed:

- ❖ The growth of the current in the coil causes a growth in the magnetic field both through and round it.
- ❖ When the current in the coil changes the magnetic field changes
- ❖ This changing magnetic field induces an e.m.f. across the coil
- ❖ The direction of the induced e.m.f. is opposite in direction to that of the current from the cell.
- ❖ For this reason it is known as a **back e.m.f.**
- ❖ The effect of this self-induced e.m.f. is to oppose the growth of current in the circuit.
- ❖ There is a delay in the current reaching its final or steady state value.
- ❖ If the switch was opened, the current would drop from its maximum value to zero. Once again a back emf is produced and the current would take longer to decay than if there was no inductor present.
- ❖ The bigger the inductor, the slower the growth in current in that circuit. (and the longer the bulb in the circuit above would take to light)

Definitions

- ❖ The magnetic inductance of a coil is directly proportional to the current

$B = L I$ where B = self inductance and I = current
The constant of proportionality is **L, the self-inductance**

- ❖ The self-induced e.m.f. is directly proportional to the rate of change of the magnetic inductance.

$$\text{e.m.f.} = - (dB / dt)$$
$$E = - (dB / dt)$$

- ❖ The **negative sign** indicates that the self-induced e.m.f. is in the opposite direction to that of the cell. (It could also be thought of as being in a direction such that it opposes the change in the current). For this reason it is known as **back e.m.f.**

$$E = - (dB / dt)$$
$$E = - L (dI / dt)$$

where:

E = self induced e.m.f. measured in Volts (V)

L = self-inductance measured in Henrys (H). This is a constant.

dI / dt = rate of change of current w.r.t. time measured in Amps per second (As^{-1})

- ❖ **L, the self-inductance** of a coil is defined as:

$$L = \frac{\text{self-induced e.m.f.}}{\text{Change in current per second}}$$

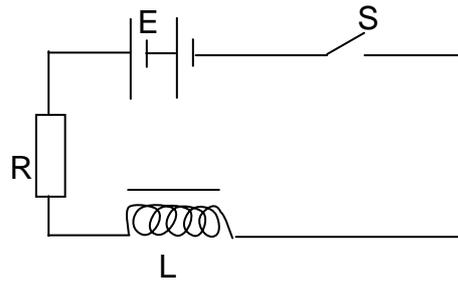
Definition of Inductance

The **inductance** of an inductor is **one Henry** if an e.m.f of **one volt** is **induced** across the ends of the inductor when the **current in the inductor changes** at a rate of **one amp per second**.

Please remember

- ❖ Coils are known as **INDUCTORS (or sometimes Chokes)** and have a property known as **INDUCTANCE**.
- ❖ Inductance depends on the design of the inductor.
Like the resistance of a resistor.
- ❖ An inductor will have a bigger inductance if it has many turns of wire, has a large area and is wound on an iron core.

What Does This Mean?



Switch S is closed at time $t = 0$

Cell e.m.f. + self induced e.m.f. due to the coil = p.d. across the resistor

$$\Rightarrow E + -L \left(\frac{dI}{dt} \right) = IR$$

$$E - L \left(\frac{dI}{dt} \right) = IR$$

when $t = 0$, $I = 0A$

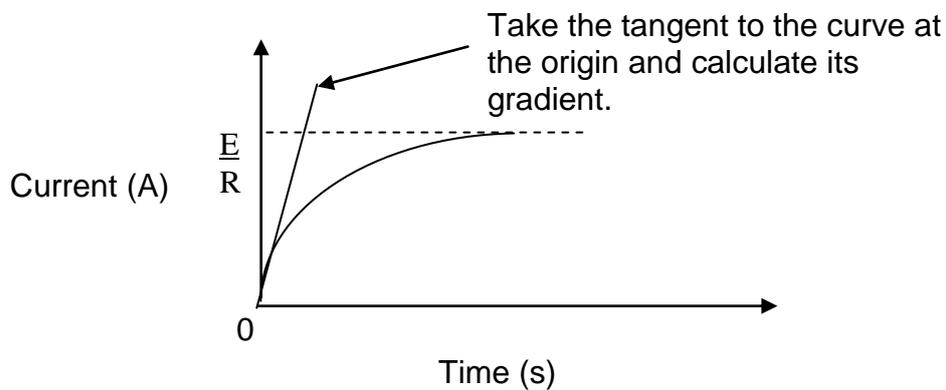
N.B. Induced e.m.f. is back e.m.f. and is therefore negative

$$\Rightarrow E - L \left(\frac{dI}{dt} \right)_{t=0} = 0$$

$$\Rightarrow L = - \frac{E}{\left(\frac{dI}{dt} \right)_{t=0}}$$

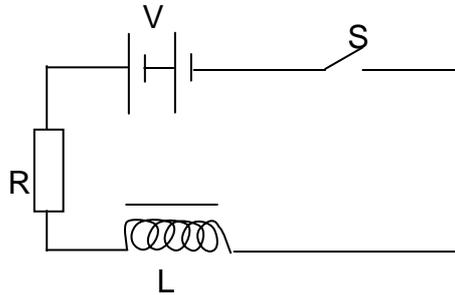
where :

$\left(\frac{dI}{dt} \right)_{t=0}$ can be found from the gradient of the current versus time graph at the point where $t = 0$ i.e. the origin



Example

Inductor in series with a resistor



1. What is the voltage across the inductor when the switch is first closed?
Numerical value is not required.
2. What is the maximum current reached in the circuit?
Numerical value is not required.
3. If the supply voltage is 3V and $R = 200\Omega$, draw a graph of current versus time for the above circuit, after the switch is closed. Assuming the resistance of the inductor is negligible.

Inductors & Energy

- ◆ Conservation of energy dictates that the direction of the induced e.m.f. must oppose the change in the current.
If it acted in the same direction as the changing current then we would be producing more current for no input of energy. The laws of physics do not allow this.
- ◆ **Lenz's Law states:**
Induced current is always in a direction to oppose the change that causes it. This is a statement of the conservation of energy because to induce a current energy has to be put into the circuit.
- ◆ The source has to do work to drive the current through the coil. It is this "work done" that appears as energy in the magnetic field of the coil. This energy can be obtained when the magnetic field collapses.
e.g. The large back e.m.f generated for a short time across a neon lamp.
- ◆ When the current is suddenly switched off large e.m.f.s are produced (and can cause sparks). At the moment of "switch off" the change in current is very large. The inductor tries to maintain the current as the magnetic field collapses and the energy stored by the magnetic field is given up.
- ◆ The energy stored in the magnetic field of an inductor can be a source of e.m.f. (a back e.m.f.). A back e.m.f. tends to be very large and only lasts for a short length of time.
- ◆ For a conductor with a current I flowing through it the energy stored is given by:

$$E = \frac{1}{2} L I^2$$

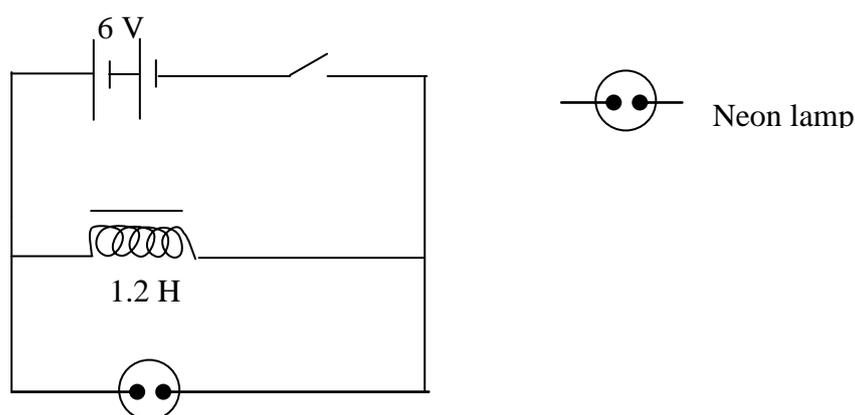
where:

E is energy measured in Joules (J)

L is self-inductance measured in Henrys (H)

I is current measured in Amps (A)

Example



The 6V supply in the circuit above is insufficient to light the neon lamp because a neon lamp needs about 80V across it before it lights.

When the switch is closed, the current builds up to its maximum value. Work done by the supply to build up the current, results in energy being stored in the magnetic field of the inductor.

If the switch is opened there is a rapid collapse of the current. The magnetic field through the inductor collapses (**changes**) to zero causing a back e.m.f. which is big enough to make the neon light flash.

If the inductor in the above example has a resistance of 4Ω **calculate the energy stored on it.**

Uses

As we now understand the principle of Lenz's Law that an induced e.m.f. always opposes the change causing it, it is easy to understand why some electrical circuits include an inductor to filter out fluctuating signals.
e.g. radios include an inductor to filter out high frequency electrical noise.

A fluorescent tube also contains an inductor because a tube needs high voltage across it to start to conduct. This high voltage is produced by generating a large e.m.f. across the inductor when the tube is switched on.

Please remember

- A pure inductor has no resistance of its own.
- A pure inductor will therefore not have a potential difference across it when the current is steady. A self-induced e.m.f. will only occur when there is a change in the current.
- A pure inductor will not dissipate energy when the current is steady. Charges pass through it with no “electrical work done”.
- The energy stored in the magnetic field around the inductor will be transformed into another energy form when the circuit is broken. i.e. it is a source of e.m.f.

Lenz’s Law tells us that:

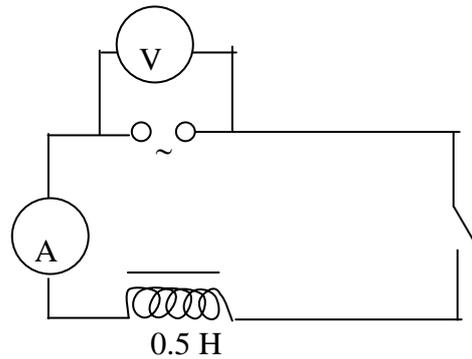
The induced e.m.f. always acts in such a direction as to opposes the change which will produce it.

Hence anything that causes a magnetic field in a coil to change will be opposed.

Do not forget that when the current decreases the effect of an inductor is to try to maintain the current

=> the induced e.m.f. will now act in the same direction as the original current.

Current and Frequency In An Inductive Circuit



Instructions

1. Set up the circuit as shown in the diagram above
2. Ensure that the supply voltage remains constant throughout the experiment
3. Vary the source frequency and record the readings of current and frequency, using a range of values from 100 Hz to 600 Hz. Use a signal generator to record the frequency and an ammeter to record the current.
4. Plot a graph of the results

Conclusion

The graph of current against $1 / \text{frequency}$ is a “best fit straight line” passing through the origin within experimental error.

This showed that the current is directly proportional to $1 / \text{frequency}$.

- ❖ The larger the frequency the smaller the current
- ❖ The larger the frequency the bigger the inductive reactance (X_L)

This opposite to the results for capacitance...where current is proportional to frequency.

- ❖ *The larger the frequency the larger the current*
- ❖ *The larger the frequency the smaller the capacitive reactance (X_C)*

Caution

- ◆ If the inductor did not have a negligible resistance then the graph of current against $(1 / \text{frequency})$ would not be a perfect straight line. A 2400 turn coil will have a resistance of about 80Ω
- ◆ The opposition to current at zero frequency is the resistance of the inductor.
- ◆ At low frequencies the current measured is a mixture of the inductive and resistive effects.

Repeat the experiment this time keeping the frequency constant and change the size of the inductor.

This experiment shows:

$$I \propto 1 / L$$

From previous experiment

$$I \propto 1 / f$$

And of course

$$I \propto V$$

Together this gives:

$$I \propto V / fL \quad \text{and the constant in the proportionality is } 2\pi$$

$$I = v / 2\pi fL = V / X_L$$

where

$$\text{Inductive Reactance } (X_L) = 2\pi fL$$

The units are ohms.

Examples

Calculate the inductive reactance of a 0.5 H inductor in a circuit supplied with 12 V a.c. at a frequency of 50 Hz.

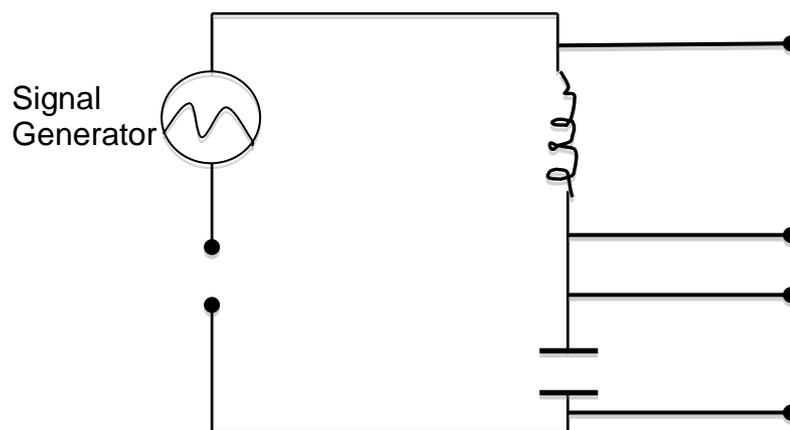
Find the voltage if the current through a 0.5 H inductor is 50 mA and the supply frequency is 120 Hz.

Summary

Frequency	Capacitive Reactance	Inductive Reactance
Low	Large	Small
High	Small	Large

Example Circuits

Using the following circuit it is possible to get a.c. and d.c. out separately.



If a high frequency is used then X_L increases ($X_L = 2\pi fL$)
Therefore the voltage across the inductor (V_{AB}) = $I X_L = V_{ac}$

Consequently the voltage across the capacitor must be d.c.

Ripple Voltage

In earlier notes on capacitors a circuit with a diode bridge and a capacitor was used to provide a rippled d.c. supply from an input a.c. supply

However the addition of a second capacitor and an inductor gives a smoothed d.c. supply output.

